DAY FOURTEEN

Properties of Matter

Learning & Revision for the Day

Elastic Behaviour

Elasticity is the property of body by virtue of which a body regains or tends to regain its original configuration (shape as well as size), when the external deforming forces acting on it, is removed.

Stress

The internal restoring force per unit area of cross-section of the deformed body is called stress.

Thus, $$\text{Stress, } \sigma = \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$$

SI unit of stress is Nm^{-2} or pascal (Pa).

Different types of stress are given below

1. Normal or Longitudinal Stress

If area of cross-section of a rod is A and a deforming force F is applied along the length of the rod and perpendicular to its cross-section, then stress produced in the rod is called normal or longitudinal stress.

$$\text{Longitudinal stress} = \frac{F_n}{A}$$

Longitudinal stress is of two types

(i) **Tensile stress** When length of the rod is increased on application of deforming force over it, then stress produced in rod is called tensile stress.

(ii) **Compressive stress** When length of the rod is decreased on application of deforming force, then the stress produced is called compressive stress.

2. Volumetric Stress

When a force is applied on a body such that it produces a change in volume and density, shape remaining same

(i) at any point, the force is perpendicular to its surface.

(ii) at any small area, the magnitude of force is directly proportional to its area.

Then, force per unit area is called volumetric stress.

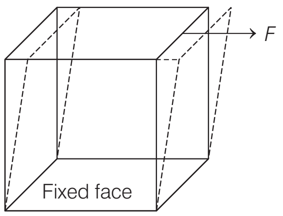
$$\therefore \text{Volumetric stress} = \frac{F_v}{A}$$

3. Shearing or Tangential Stress

When the force is applied tangentially to a surface, then it is called tangential or shearing stress.

$$\text{Tangential stress} = \frac{F_t}{A}$$

It produces a change in shape, volume remaining same.



An object under tangential deforming forces

Strain

Strain is the ratio of change in configuration to the original configuration of the body.

Being the ratio of two similar quantities, strain is a unitless and dimensionless quantity.

(i) When the deforming force causes a change in length, it is called **longitudinal strain**. For a wire or rod, longitudinal strain is defined as the ratio of change in length to the original length.

$$\therefore \text{Longitudinal strain} = \frac{\text{Change in length } (\Delta L)}{\text{Original length } (L)}$$

(ii) When the deforming force causes a change in volume, the strain is called **volumetric strain**.

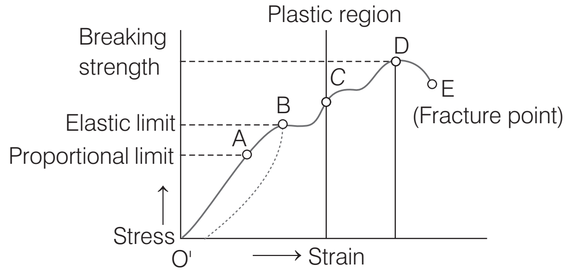
$$\text{Volumetric strain} = \frac{\text{Change in volume } (\Delta V)}{\text{Original volume } (V)}$$

(iii) When the deforming force, applied tangentially to a surface, produces a change in shape of the body, the strain developed is called **shearing strain** or **shear**.

$$\text{Shearing strain, } \phi = \frac{x}{L}$$

Stress-Strain Relationship

For a solid, the graph between stress (either tensile or compressive) and normal strain is shown in figure.



A typical stress-strain curve

- In the above graph, point A is called **proportional limit**. Till this point, stress and strain are proportional to each other.
- From point A to B, stress and strain are not proportional, B is called **elastic limit** and OB is **elastic region**.
- Beyond point B, strain increases without increase in stress, it is called **plastic behaviour**. Region between point C and D is called **plastic region**.
- Finally, at point D, wire may break, maximum stress corresponding to point D is called **breaking stress**.

The materials of the wire, which break as soon as stress is increased beyond the elastic limit are called **brittle**. Graphically, for such materials the portion of graph between B and E is almost zero. While the materials of the wire, which have a good plastic range (portion between B and E) are called **ductile**.

Hooke's Law

According to the Hooke's law, for any body, within the elastic limit, stress developed is directly proportional to the strain produced.

$$\text{Stress} \propto \text{Strain}$$

$$\text{Stress} = E \times \text{Strain}$$

The ratio of stress to strain, within the elastic limit, is called the **coefficient (or modulus) of elasticity** for the given material.

Depending on the type of stress applied and resulting strain, we have the following three of elasticity given as,

$$E = \frac{\text{Stress}}{\text{Strain}}$$

There are three modulus of elasticity.

1. Young's Modulus

Young's modulus of elasticity (Y) is defined as the ratio of normal stress (either tensile or compressive stress) to the longitudinal strain within a elastic limit.

Young's modules,

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

2. Bulk Modulus

It is defined as the ratio of the normal stress to the volumetric strain. Coefficient of volume elasticity,

$$B = \frac{F/A}{\Delta V/V} = -\frac{pV}{\Delta V}$$

where, $p = \frac{F}{A}$ = the pressure or stress negative sign signifies that for an increase in pressure, the volume will decrease.

Reciprocal of bulk modulus is called **compressibility.**

3. Modulus of Rigidity (Shear modulus)

It is defined as the ratio of tangential stress to shearing stress.

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}} = \frac{F/A}{\phi} = \frac{F}{A\phi} = \frac{FL}{Ax}$$

- **Breaking force** depends upon the area of cross-section of the wire.

  $\therefore$ Breaking force $\propto A$

  Breaking force $= P \times A$

  Here, P is a constant of proportionality and known as breaking stress.

Poisson's Ratio

For a long bar, the Poisson's ratio is defined as the ratio of lateral strain to longitudinal strain.

$$\therefore \text{ Poisson's ratio, } \sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{\Delta D/D}{\Delta L/L} = \frac{\Delta r/r}{\Delta L/L}$$

Poisson's ratio is a unitless and dimensionless term. Its value depends on the nature of the material. Theoretically, value of σ must lie between -1 and $+0.5$ but for most metallic solids $0 < \sigma < 0.5$.

Inter-Relations Between Elastic Constants

Y = Young's modulus, η = Rigidity modulus,

B = Bulk modulus, σ = Poisson's ratio

The inter relation between elastic constants are

$$Y = 2\eta(1+\sigma), \quad Y = 3B(1-2\sigma)$$

$$\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{B} \quad \text{or} \quad Y = \frac{9B\eta}{\eta + 3B} \quad \text{or} \quad \sigma = \frac{3B - 2\eta}{6B + 2\eta}$$

Work Done (or Potential Energy Stored) in a Stretched Wire

Work is done against the internal restoring forces, while stretching a wire. This work is stored as elastic potential energy. The work done is given by

- Work done $W = \frac{1}{2} \times$ stretching force $\times$ elongation

  $$= \frac{1}{2} F\Delta L = \frac{1}{2}\frac{YA}{L}(\Delta L)^2$$

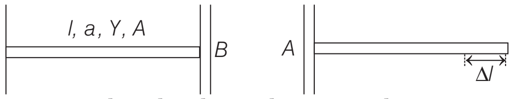
  = Energy stored in the wire (U)

- Energy stored per unit volume (or energy density)

  $$= \frac{U}{V} = \frac{1}{2}\frac{F\Delta L}{AL} = \frac{1}{2}\,\text{stress} \times \text{strain} = \frac{Y}{2}(\text{strain})^2$$

Thermal Stresses and Strains

When a body is allowed to expand or contract with increase or decrease in temperature, no stresses are induced in the body. But if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are called thermal stresses or temperature stresses. The corresponding strains are called thermal strains or **temperature strains**.



A rod under thermal stress and strain

By definition, coefficient of linear expansion $\alpha = \frac{\Delta l}{l\theta}$

thermal strain $\frac{\Delta l}{l} = \alpha\Delta\theta$

So thermal stress $= Y\alpha\Delta\theta$

Tensile or compressive force produced in the body

$$F = YA\alpha\Delta\theta$$

Fluid Statics

The substances which flow are called fluids, that includes both liquid and gas. The science of fluids at rest is called fluid statics where fluid mass is stationary w.r.t. container, containing the fluid.

Thrust and Pressure

Normal force exerted by fluid at surface in contact is called **thrust** of fluid.

The thrust exerted by a fluid at rest per unit surface area of contact surface is called the **fluid pressure**.

$$\therefore \quad \text{Pressure } p = \frac{\text{Normal force (thrust) } F}{\text{Surface area } A}$$

Pressure is a scalar and its SI unit is Nm^{-2} or pascal (Pa), where, 1 Pa = 1 N m^{-2}.

Pressure due to Fluid Column

Pressure p exerted by the fluids at the bottom of container having height h

$$p = h\rho g$$

where, ρ = density of fluid.

Gauge Pressure

The pressure difference between the real hydrostatic pressure and the atmospheric pressure is known as the gauge pressure.

$\therefore$ Gauge pressure = Real pressure (p) – Atmospheric pressure (p_0)

Pascal's Law

According to Pascal's law of transmission of pressure, the increase in pressure at any one point of the enclosed liquid in equilibrium or at rest, is transmitted equally to all other points of the liquid and also to the walls of the container. Hydraulic lift, hydraulic press, hydraulic brakes etc. are based on the Pascal's law.

Archimedes' Principle and Buoyancy

When a body is immersed in a fluid, it experiences an upthrust due to the fluid and as a result the apparent weight of the body is reduced.

∴ Apparent weight of the body

= weight of the body − upthrust due to fluid

= weight of the body − weight of the fluid displaced

e.g. For a floating body, the volume of a body $(V - V_s)$ remaining outside the liquid will be given by

$$V_0 = V - V_s = V - V\frac{\rho}{\sigma} = V\left(1 - \frac{\rho}{\sigma}\right)$$

where, ρ = density of liquid

and σ = density of body immersed in liquid

Buoyant Force or Buoyancy

Buoyant force, $F = h\rho g A = mg$

where, h = height of body immersed in liquid,

m = mass of body and A = area

- It is an upward force acting on the body immersed in a liquid.
- It is equal to the weight of liquid displaced by the immersed part of the body.
- The buoyant force acts at the centre of buoyancy which is the centre of gravity of the liquid displaced by the body when immersed in the liquid.
- The line joining the centre of gravity and centre of buoyancy is called central line.
- Metacentre, is a point where the vertical line passing through the centre of buoyancy intersects the central line.

Laws of Floatation

When a body of density ρ_B and volume V is immersed in a liquid of density σ, the forces acting on the body are

- The weight of body $W = mg = V\rho_B g$ acting vertically downwards through the centre of gravity of the body.
- The upthrust $F = V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid *i.e.,* centre of buoyancy.

So, the following three cases are possible.

Case I The density of body is greater than that of liquid (i.e. $\rho_B > \sigma$). In this case, as weight will be more than upthrust, the body will sink. $(W > F)$

Case II The density of body is equal to the density of liquid (i.e. $\rho_B = \sigma$). In this case, $W = F$. so, the body will float fully submerged in neutral equilibrium anywhere in the liquid.

Case III The density of body is less than that of liquid (i.e. $\rho_B < \sigma$). In this case, $W < F$, so the body will move upwards and **in equilibrium** will float partially immersed in the liquid such that

$$W = V_{\text{in}}\sigma g$$

[V_{in} is the volume of body in the liquid]

or $V\rho_B g = V_{\text{in}}\rho g$ [as, $W = mg = \rho_B V g$]

or $V\rho_B = V_{\text{in}}\sigma$ ...(i)

Viscosity

Viscosity is the property of a fluid due to which it opposes the relative motion between its different layers.

Force between the layers opposing the relative motion is called **viscous force**.

If there are two fluid layers having surface area A and velocity gradient $d\mathbf{v}/dr$, then the viscous force is given by $\mathbf{F} = -\eta A\frac{d\mathbf{v}}{dr}$. where, constant η is called the coefficient of viscosity of the given fluid.

SI unit of coefficient of viscosity is N m^{-2}s or Pa-s or poiseuille (Pl).

Terminal Velocity

If a small spherical body is dropped in a fluid, then initially it is accelerated under the action of gravity. However, with an increase in speed, the viscous force increases and soon it balances the weight of the body.

Now, the body moves with a constant velocity, called the **terminal velocity**.

Terminal velocity v_t is given by

$$v_t = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

where, r = radius of the falling body,

ρ = density of the falling body

and σ = density of the fluid.

Stokes' Law

Stokes proved that for a small spherical body of radius r moving with a constant speed v called terminal velocity through a fluid having coefficient of viscosity η, the viscous force F is given by

$$F = 6\pi\eta r v$$

It is known as Stokes' law.

Streamline and Turbulent Flow

Flow of a fluid is said to be **streamlined** if each element of the fluid passing through a particular point travels along the same path, with exactly the same velocity as that of the preceding element. A special case of streamline flow is **laminar flow**.

A **turbulent flow** is the one in which the motion of the fluid particles is disordered or irregular.

Critical Velocity

For a fluid, the critical velocity is that limiting velocity of the fluid flow upto which the flow is streamlined and beyond which the flow becomes turbulent. Value of critical velocity for the flow of liquid of density ρ and coefficient of viscosity η, flowing through a horizontal tube of radius r is given by

$$v_c \propto \frac{\eta}{\rho r}$$

where, r = radius of tube.

Reynold's Number (N_R)

Reynold's number as the ratio of the inertial force per unit area to the viscous force per unit area for a fluid.

$$N_R = \frac{v^2\rho}{\eta v/r} = \frac{\rho v r}{\eta}$$

A smaller value of Reynold's number (generally $N_R \le 1000$) indicates a streamline flow but a higher value ($N_R \ge 1500$) indicates that the flow is turbulent and between 1000 to 1500, the flow is unstable.

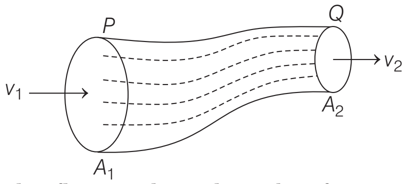
Equation of Continuity

Let us consider the streamline flow of an ideal, non-viscous fluid through a tube of variable cross-section.

Let at the two sections, the cross-sectional areas be A_1 and A_2, respectively and the fluid flow velocities are v_1 and v_2, then according to the equation of continuity

$$A_1 v_1 \rho_1 = A_2 v_2 \rho_2$$

where, ρ_1 and ρ_2 are the respective densities of the fluid. Equation of continuity is based on the conservation of mass.



A liquid is flowing through a tube of non-uniform cross-section

If the fluid which is flowing, is incompressible, then

$$\rho_1 = \rho_2$$

So, equation of continuity is simplified as

$$A_1 v_1 = A_2 v_2 \quad \text{or} \quad Av = \text{constant}$$

Energy of a Flowing Liquid

There are three types of energies in a flowing liquid.

- **Pressure Energy** If p is the pressure on the area A of a fluid and the liquid moves through a distance l due to this pressure, then

  Pressure energy of liquid = work done

  $$= \text{force} \times \text{displacement} = pAl$$

  The volume of the liquid is Al. Hence, pressure energy per unit volume of liquid $= \frac{pAl}{Al} = p$

- **Kinetic Energy** If a liquid of mass m and volume V is flowing with velocity v, then the kinetic energy $= \frac{1}{2}mv^2$

  ∴ Kinetic energy per unit volume of liquid

  $$= \frac{1}{2}\left(\frac{m}{V}\right)v^2 = \frac{1}{2}\rho v^2$$

  Here, ρ is the density of liquid and V = volume.

- **Potential Energy** If a liquid of mass m is at a height h from the reference line ($h = 0$), then its potential energy is mgh.

  ∴ Potential energy per unit volume of the liquid

  $$= \left(\frac{m}{V}\right)gh = \rho gh$$

Bernoulli's Principle

According to the Bernoulli's principle for steady flow of an incompressible, non-viscous fluid through a tube/pipe, the total energy (i.e. the sum of kinetic energy, potential energy and pressure energy) per unit volume (or per unit mass too) remains constant at all points of flow provided that there is no source or sink of the fluid along the flow.

Mathematically, we have

$$p + \rho gh + \frac{1}{2}\rho v^2 = \text{constant} \quad \text{or} \quad \frac{p}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$$

In this expression, $\frac{v^2}{2g}$ is **velocity head** and $\frac{p}{\rho g}$ is **pressure head**.

Velocity of Efflux

- If a liquid is filled in a vessel up to a height H and a small orifice O is made at a height h, then from Bernoulli's theorem it can be shown that velocity of efflux v of the liquid from the vessel is

  $$v = \sqrt{2g(H - h)}$$

- The flowing fluid describes a parabolic path and hits the base level at a horizontal distance (called the **range**) $R = 2\sqrt{h(H - h)}$.

  The range is maximum, when $h = \frac{H}{2}$ and in that case $R_{max} = H$.

Applications Based on the Bernoulli's Principle

- The action of carburetor, paintgun, scent sprayer, atomiser and insect sprayer is based on the Bernoulli's principle.
- The action of the Bunsen's burner, gas burner, oil stove and exhaust pump is also based on the Bernoulli's principle.
- Motion of a spinning ball (Magnus effect) is based on Bernoulli's theorem.
- Blowing of roofs by wind storms, attraction between two parallel moving boats moving close to each other, fluttering of a flag etc., are also based on Bernoulli's theorem.

Surface Tension

Surface tension is the property of a liquid due to which its free surface behaves like a stretched elastic membrane and tends to have the least possible surface area.

Surface tension $S = \frac{\text{Force}}{\text{Length}} = \frac{F}{l}$

Here, F is force acting on the unit length of an imaginary line drawn of the surface of the liquid.

SI unit of surface tension is Nm^{-1} or Jm^{-2}. It is a scalar and its dimensional formula is $[MT^{-2}]$.

Surface Energy

Surface energy of a liquid is the potential energy of the molecules of a surface film of the liquid by virtue of its position.

When the surface area of a liquid is increased, work is done against the cohesive force of molecules and this work is stored in the form of additional surface energy.

Increase in surface potential energy

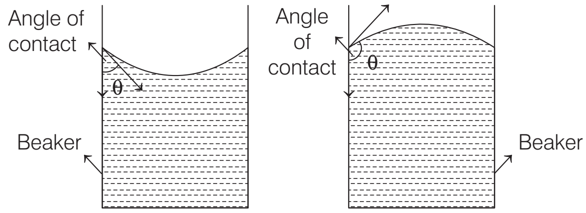
$$\Delta U = \text{Work done } (\Delta W) = S\Delta A$$

where, ΔA is the increase in surface area of the liquid.

- **Work done in Blowing a Liquid Drop** If a liquid drop is blown up from a radius r_1 to r_2, then work done in the process,
$$W = S(A_2 - A_1) = S \times 4\pi\ (r_2^2 - r_1^2)$$
- **Work done in Blowing a Soap Bubble** As a soap bubble has two free surfaces, hence, work done in blowing a soap bubble, so as to increase its radius from r_1 to r_2, is given by
$$W = S \times 8\pi\ (r_2^2 - r_1^2)$$
- **Work done in Splitting a Bigger Drop into *n* Smaller Droplets** If a liquid drop of radius R is split up into n smaller droplets, all of the same size, then radius of each droplet
$$r = R(n)^{-1/3}$$
and work done
$$W = S \times 4\pi\,(nr^2 - R^2) = S \times 4\pi R^2\ (n^{1/3} - 1)$$
- **Coalescence of Drops** If n small liquid drops of radius r each, combine together so as to form a single bigger drop of radius $R = n^{1/3} r$ then in the process, energy is released. Release of energy is given by
$$\Delta U = S \times 4\pi\,(nr^2 - R^2) = S \times 4\pi r^2 n\,(1 - n^{-1/3})$$

Angle of Contact

Angle of contact for a given liquid-solid combination is defined as the angle subtended between the tangents to the liquid surface and the solid surface, inside the liquid, the tangents are drawn at the point of contact.



Shape of meniscus for different media

- Value of the angle of contact depends on the nature of liquid and solid both.
- For a liquid having concave meniscus when adhesive force > cohesive force angle of contact θ is acute $(\theta < 90°)$ but for a convex meniscus (when cohesive force > adhesive force) the angle of contact is obtuse $(\theta > 90°)$.
- Value of angle of contact θ decreases with an increase in temperature.

Excess Pressure Over a Liquid Film

If a free liquid surface film is plane, then pressure on the liquid and the vapour sides of the film are the same, otherwise there is always some pressure difference. Following cases arise.

- For a spherical liquid drop of radius r, the excess pressure inside the drop $p = \frac{2S}{r}$

  where, S = surface tension of the liquid.
- For an air bubble in a liquid, excess pressure $p = \frac{2S}{r}$
- For a soap bubble in air, excess pressure $p = \frac{4S}{r}$

Capillary Rise or Capillarity

Capillarity is the phenomenon of rise or fall of a liquid in a capillary tube as compared to that in a surrounding liquid.

The height h up to which a liquid will rise in a capillary tube is given by

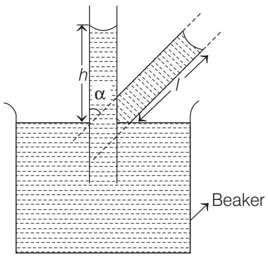
$$h = \frac{2S\cos\theta}{r\rho g} = \frac{2S}{R\rho g}$$

where, r = radius of the capillary tube

and $R = \frac{r}{\cos\theta}$ = radius of liquid meniscus.

The rise in capillary tube $h \propto \frac{1}{r}$ (Jurin's law).

- If a capillary tube, dipped in a liquid is tilted at an angle α from the vertical, the vertical height h of the liquid column remains the same. However, the length of the liquid column (l) in the capillary tube increases to



Effect of tilting capillary tube in a liquid

$$l = \frac{h}{\cos \alpha}.$$

- If the capillary tube is of insufficient length, the liquid rises up to the upper end of the tube and then the radius of its meniscus changes from R to R' such that $hR = h'R'$, where h' = insufficient length of the tube.
- After connection due to the weight of liquid contained in the meniscus, the formula for the height is given by

$$h = \frac{2s}{\rho r g} - \frac{r}{3}$$

This is known as **ascent formula**.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of → **JEE Main 2017 (Offline)**

(a) $\frac{1}{9}$ (b) 81
(c) $\frac{1}{81}$ (d) 9

2 The Young's modulus of brass and steel are respectively 1.0×10^{11} Nm^{-2} and 2.0×10^{11} Nm^{-2}. A brass wire and a steel wire of the same length extend by 1 mm, each under the same force. If radii of brass and steel wires are R_B and R_S respectively, then

(a) $R_S = \sqrt{2}\, R_B$ (b) $R_S = \frac{R_B}{\sqrt{2}}$
(c) $R_S = 4R_B$ (d) $R_S = \frac{R_B}{2}$

3 One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(a) 0.25 (b) 0.50
(c) 2.00 (d) 4.00

4 The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied ?

(a) Length = 50 cm, diameter = 0.5 mm
(b) Length = 100 cm, diameter = 1 mm
(c) Length = 200 cm, diameter = 2 mm
(d) Length = 300 cm, diameter = 3 mm

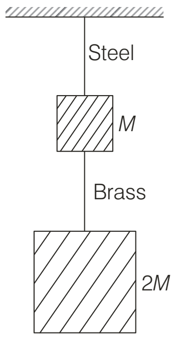
5 The length of a metal wire is l_1 when the tension in it is T_1 and is l_2 when the tension is T_2. The original length of the wire is

(a) $\frac{l_1 + l_2}{2}$ (b) $\frac{l_1T_2 + l_2T_1}{T_1 + T_2}$
(c) $\frac{l_1T_2 - l_2T_1}{T_2 - T_1}$ (d) $\sqrt{T_1T_2l_1l_2}$

6 Two wires are made of the same material and have the same volume. However, wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount? → **AIEEE 2009**

(a) F (b) $4F$ (c) $6F$ (d) $9F$

7 If the ratio of lengths, radii and Young's modulii of steel and brass wires in the figure are a, b and c respectively, then the corresponding ratio of increase in their lengths is → **JEE Main (Online) 2013**



(a) $\frac{3c}{2ab^2}$ (b) $\frac{2a^2c}{b}$ (c) $\frac{3a}{2b^2c}$ (d) $\frac{2ac}{b^2}$

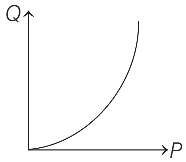
8 The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change in volume is 10% keeping temperature constant. The bulk modulus of the medium is

(a) 204.8×10^5 Pa (b) 102.4×10^5 Pa
(c) 51.2×10^5 Pa (d) 1.55×10^5 Pa

9 A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$ is **→ JEE Main 2018**

(a) $\frac{Ka}{mg}$ (b) $\frac{Ka}{3mg}$
(c) $\frac{mg}{3Ka}$ (d) $\frac{mg}{Ka}$

10 The graph shows the behaviour of a length of wire in the region for which the substance obeys Hooke's law. P and Q represent



(a) P = applied force, Q = extension
(b) P = extension, Q = applied force
(c) P = extension, Q = stored elastic energy
(d) P = stored elastic energy, Q = extension

11 If work done in stretching a wire by 1mm is 2 J. The work necessary for stretching another wire of same material but with double the radius and half the length, by 1 mm distance, is

(a) 16 J (b) 4 J
(c) 1/4 J (d) 8 J

12 Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's modulii Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to

(a) 2 : 3 (b) 1 : 1
(c) 3 : 2 (d) 4 : 9

13 The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is
(For steel, Young's modulus is 2×10^{11} Nm^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1}) **→ JEE Main 2014**

(a) 2.2×10^8 Pa (b) 2.2×10^9 Pa
(c) 2.2×10^7 Pa (d) 2.2×10^6 Pa

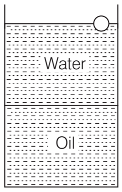
14 A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by t°C, the linear stress developed in it is **→ AIEEE 2011**

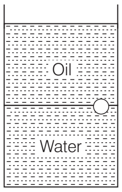
(a) $\frac{\alpha t}{Y}$ (b) $\frac{Y}{\alpha t}$ (c) $Y\alpha t$ (d) $\frac{1}{Y\alpha t}$

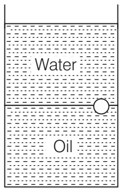
15 A wooden block of mass m and density ρ is tied to a string, the other end of the string is fixed to the bottom of a tank. The tank is filled with a liquid of density σ with $\sigma > \rho$. The tension in the string will be

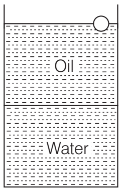
(a) $\left(\frac{\sigma - \rho}{\sigma}\right) mg$ (b) $\left(\frac{\sigma - \rho}{\rho}\right) mg$
(c) $\frac{\rho\, mg}{\sigma}$ (d) $\frac{\sigma\, mg}{\rho}$

16 A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position? **→ AIEEE 2010**

(a)


(b)


(c)


(d)


17 Two mercury drops (each of radius r) merge to form a bigger drop. The surface energy of the bigger drop, if T is the surface tension, is **→ AIEEE 2011**

(a) $2^{5/3} \pi^2 T$ (b) $4\pi^2 T$
(c) $2\pi r^2 T$ (d) $2^{8/3} \pi r^2 T$

18 A raindrop of radius 0.2 cm is falling through air with a terminal velocity of 8.7 m/s. The viscosity of air in SI units is (take, $\rho_{water} = 1000$ kg/m^3 and $\rho_{air} = 1$ kg/m^3).

(a) 10^{-4} poise (b) 1×10^{-3} poise
(c) 8.6×10^{-3} poise (d) 1.02×10^{-3} poise

19 If a ball of steel (density $\rho = 7.8$ g cm^{-3}) attains a terminal velocity of 10 cms^{-1} when falling in a tank of water (coefficient of viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa-s) then its terminal velocity in glycerine ($\rho = 1.2$ gcm^{-3}, $\eta = 13.2$ Pa-s) would be nearly **→ AIEEE 2011**

(a) 1.6×10^{-5} cms^{-1} (b) 6.25×10^{-4} cms^{-1}
(c) 6.45×10^{-4} cms^{-1} (d) 1.5×10^{-5} cms^{-1}

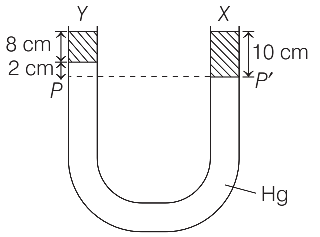
20 Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to
→ AIEEE 2011

(a) 7.5×10^{-3} m (b) 9.6×10^{-3} m
(c) 3.6×10^{-3} m (d) 5.0×10^{-3} m

21 At what speed, the velocity head of water is equal to pressure head of 40 cm of Hg?

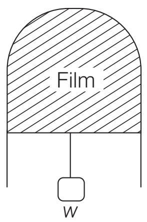
(a) 10.3 ms^{-1} (b) 2.8 ms^{-1}
(c) 5.6 ms^{-1} (d) 8.4 ms^{-1}

22 A liquid *X* of density 3.36 g/cm^3 is poured in a U-tube, which contains Hg. Another liquid *Y* is poured in the left arm with height 8 cm and upper levels of *X* and *Y* are same. What is the density of *Y*?



(a) 0.8 g /cc (b) 1.2 g /cc (c) 1.4 g /cc (d) 1.6 g /cc

23 A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2}N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is



→ AIEEE 2012

(a) 0.0125 Nm^{-1} (b) 0.1 Nm^{-1}
(c) 0.05 Nm^{-1} (d) 0.025 Nm^{-1}

24 Work done in increasing the size of a soap bubble from radius of 3 cm to 5 cm is nearly (surface tension of soap solution = 0.03 Nm^{-1})
→ AIEEE 2011

(a) 0.2 π mJ (b) 2 π mJ (c) 0.4 π mJ (d) 4 π mJ

25 While measuring surface tension of water using capillary rise method, height of the lower meniscus from free surface of water is 3 cm while inner radius of capillary tube is found to be 0.5 cm. Then, compute tension of water using this data (take, contact angle between glass and water as 0 and $g = 9.81 \, m/s^2$).

(a) 0.72 N/m
(b) 0.77 N/m
(c) 1.67 N/m
(d) None of the above

26 Match the following columns.

	Column I		Column II
A.	Magnus energy	1.	Pascal's law
B.	Loss of energy	2.	Bernoulli's principle
C.	Pressure is same at the same level in a liquid	3.	Viscous force
D.	Gas burner	4.	Spinning ball

Codes

	A	B	C	D		A	B	C	D
(a)	1	4	2	3	(b)	1	2	3	4
(c)	2	2	4	3	(d)	4	3	1	2

Direction (Q. Nos. 27-30) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

27 **Statement I** Aeroplanes are made to run on the runway before take off, so that they acquire the necessary lift.

Statement II This is as per the Bernoulli's theorem.

28 **Statement I** Finer the capillary, greater is the height to which the liquid rises in the tube.

Statement II This is in accordance with the ascent formula.

29 **Statement I** The bridges are declared unsafe after a long use.

Statement II Elastic strength of bridges decreases with time.

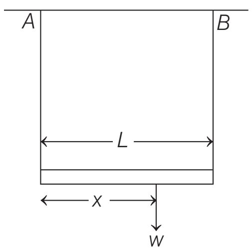
30 **Statement I** A small drop of mercury is spherical but bigger drops are oval in shape.

Statement II Surface tension of liquid decreases with an increase in temperature.

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A light rod of length L is suspended from a support horizontally by means of two vertical wires A and B of equal lengths as shown in the figure. Cross-section area of A is half that of B and Young's modulus of A is double than that of B. A weight w is hung on the rod as shown. The value of x, so that the stress in A is same as that in B, is



(a) $\frac{L}{3}$ (b) $\frac{L}{2}$ (c) $\frac{2L}{3}$ (d) $\frac{3L}{4}$

2 A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y, then $\frac{1}{Y}$ is equal to (take, g = gravitational acceleration)

➞ **JEE Main 2015**

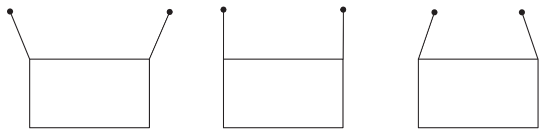
(a) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ (b) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

(c) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (d) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

3 The length of an elastic string is 1m, when the longitudinal tension is 4 N and the length is b metres, when the tension is 5 N. The length of the string (in metre) when the longitudinal tension is 9 N is

(a) $2b - \frac{a}{2}$ (b) $5b - 4a$ (c) $4a - 3b$ (d) $a - b$

4 A rectangular frame is to be suspended symmetrically by two strings of equal length on two supports (figure). It can be done in one of the following three ways:



(a) (b) (c)

The tension in the strings will be

(a) the same in all cases (b) least in (a)
(c) least in (b) (d) least in (c)

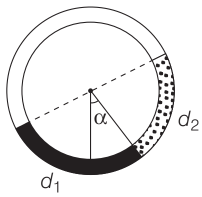
5 To what depth must a rubber ball be taken in deep sea, so that its volume is decreased by 0.1%. (The bulk modulus of rubber is 9.8×10^8 N/m^2; and the density of sea water is 10^3 kg/m^3.)

(a) 100 m (b) 60 m (c) 75 m (d) 65 m

6 The free surface of oil in a tanker, at rest, is horizontal. If the tanker starts accelerating the free surface will be tilted by an angle θ. If the acceleration is a ms^{-2}, then the slope of the free surface is

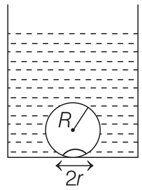
(a) g/a (b) a/g (c) ag (d) a^2/g

7 There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio d_1 / d_2 is ➞ **JEE Main 2014**



(a) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (b) $\frac{1+\cos\alpha}{1-\cos\alpha}$ (c) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (d) $\frac{1+\sin\alpha}{1-\cos\alpha}$

8 On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$ and the surface tension of water is T, value of r just before bubbles detach is (density of water is ρ) ➞ **JEE Main 2014**



(a) $R^2\sqrt{\frac{2\rho_w g}{3T}}$ (b) $R^2\sqrt{\frac{\rho_w g}{6T}}$ (c) $R^2\sqrt{\frac{\rho_w g}{T}}$ (d) $R^2\sqrt{\frac{3\rho_w g}{T}}$

9 An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg] ➞ **JEE Main 2014**

(a) 16 cm (b) 22 cm (c) 38 cm (d) 6 cm

10 Two narrow bores of diameter 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} N/m. Take the angle of contact to be zero and density of water to be 1.0×10^3 kg/m^3.(take, g = 9.8 m/s^2).

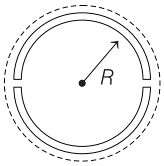
(a) 6 mm (b) 2 mm (c) 5 mm (d) 3 mm

11 A long metal rod of length l and relative density σ, is held vertically with its lower end just touching the surface of water. The speed of the rod when it just sinks in water, is given by

(a) $\sqrt{2gl\sigma}$ (b) $\sqrt{2gl(2\sigma - 1)}$

(c) $\sqrt{2gl\left(1-\frac{1}{2\sigma}\right)}$ (d) $\sqrt{2gl}$

12 A wooden wheel of radius R is made of two semi-circular parts (see figure). The two parts are held together by a ring made of a metal strip of cross-sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semi-circular parts together. If the coefficient of linear expansion of the metal is α and its Young's modulus is Y, the force that one part of the wheel applies on the other part is **→ AIEEE 2012**

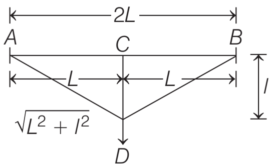


(a) $2\pi SY\alpha\Delta T$ (b) $SY\alpha\Delta T$ (c) $\pi SY\alpha\Delta T$ (d) $2SY\alpha\Delta T$

13 A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel, if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2, respectively? **→ JEE Main 2013**

(a) 188.5 Hz (b) 178.2 Hz (c) 200.5 Hz (d) 770 Hz

14 A wire of length $2L$ and radius r is stretched and clamped between A and B. If the Young's modulus of the material of the wire be Y, tension in the wire, when stretched in the position ADB will be



(a) $\pi r^2 YLt$ (b) $\pi r^2 Yl^2/2L^2$
(c) $2\pi r^2 YL^2/l^2$ (d) None of these

15 A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length veritcal from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is **→ JEE Main 2013**

(a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k}\left(\frac{1-LA\sigma}{M}\right)$
(c) $\frac{Mg}{k}\left(\frac{1-LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k}\left(\frac{1+LA\sigma}{M}\right)$

16 A uniform wire (Young's modulus 2×10^{11} Nm^{-2}) is subjected to longitudinal tensile stress of 5×10^7 Nm^{-2}. If the overall volume change in the wire is 0.02%, the fractional decrease in the radius of the wire is close to **→ JEE Main (Online) 2013**

(a) 1.0×10^{-4} (b) 1.5×10^{-4} (c) 0.17×10^{-4} (d) 5×10^{-4}

17 A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional areas are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm.

If the Young's modulii of copper and steel are respectively 1.0×10^{11} Nm^{-2} and 2.0×10^{11} Nm^{-2}, the total extension of the composite wire is **→ JEE Main (Online) 2013**

(a) 1.75 mm (b) 2.0 mm (c) 1.50 mm (d) 1.25 mm

18 A wire of mass m, and length l is suspended from a ceiling. Due to its own weight it elongates, consider cross-section area of wire as A and Young's modulus of material of wire as Y. The elongation in the wire is

(a) $\frac{2mg}{3YA}$ (b) $\frac{mgl}{YA}$ (c) $\frac{mgl}{2YA}$ (d) $\frac{3mg}{YAl}$

Direction (Q. Nos. 19-20) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

19 **Statement I** In taking into account the fact that any object which floats must have an average density less than that of water, during world war-I, a number of cargo vessels were made of concrete.

Statement II Concrete cargo vessels were filled with air.

20 **Statement I** The stream of water flowing at high speed from a garden hosepipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Statement II In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

ANSWERS

SESSION 1	**1** (d)	**2** (b)	**3** (c)	**4** (a)	**5** (c)	**6** (d)	**7** (c)	**8** (d)	**9** (c)	**10** (c)
	11 (a)	**12** (c)	**13** (a)	**14** (c)	**15** (b)	**16** (b)	**17** (d)	**18** (b)	**19** (b)	**20** (c)
	21 (b)	**22** (a)	**23** (d)	**24** (c)	**25** (b)	**26** (d)	**27** (a)	**28** (a)	**29** (a)	**30** (b)
SESSION 2	**1** (c)	**2** (a)	**3** (b)	**4** (c)	**5** (a)	**6** (b)	**7** (c)	**8** (a)	**9** (a)	**10** (c)
	11 (c)	**12** (d)	**13** (b)	**14** (b)	**15** (c)	**16** (c)	**17** (d)	**18** (c)	**19** (b)	**20** (a)

Hints and Explanations

SESSION 1

1 $\because$ Stress $= \dfrac{\text{Weight}}{\text{Area}}$

Volume will become (9^3) times.

So weight = volume × density × g will also become $(9)^3$ times.

Area of cross-section will become $(9)^2$ times

$$= \frac{9^3 \times W_0}{9^2 \times A_0} = 9\left(\frac{W_0}{A_0}\right)$$

Hence, the stress increases by a factor of 9.

2 $\Delta L = \dfrac{FL}{YA} = \dfrac{FL}{Y\pi R^2}$

As F, L and ΔL are the same, hence

$$YR^2 = \text{constant}$$

$\therefore\ 2.0\times 10^{11} R_S^2 = 1.0\times 10^{11} R_B^2$

$\Rightarrow\ R_S = \dfrac{R_B}{\sqrt{2}}$

3 $\Delta l = \dfrac{FL}{AY} = \dfrac{FL}{(\pi r^2)Y}$

$\therefore\ \Delta l \propto \dfrac{L}{r^2}$

$\therefore\ \dfrac{\Delta l_1}{\Delta l_2} = \dfrac{L/R^2}{2L/(2R)^2} = 2$

4 $\Delta l = \dfrac{Fl}{AY} = \dfrac{Fl}{\left(\dfrac{\pi d^2}{4}\right)Y}$ or $(\Delta l) \propto \dfrac{l}{d^2}$

Now, $\dfrac{l}{d^2}$ is maximum in option (a).

5 Let l = original length of the wire,

$$\Delta l_1 = l_1 - l$$

Similarly, the change in length of the second wire is

$$\Delta l_2 = l_2 - l$$

Now, $Y = \dfrac{T_1}{A} \times \dfrac{l}{\Delta l_1} = \dfrac{T_2}{A} \times \dfrac{l}{\Delta l_2}$

$\Rightarrow\ \dfrac{T_1}{\Delta l_1} = \dfrac{T_2}{\Delta l_2}$

$\Rightarrow\ \dfrac{T_1}{l_1 - l} = \dfrac{T_2}{l_2 - l}$

$\Rightarrow\ T_1 l_2 - T_1 l = T_2 l_1 - T_2 l$

$\Rightarrow\ l = \dfrac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$

6 As volume is same.

$\therefore\ A_1 l_1 = A_2 l_2$

$\Rightarrow\ l_2 = \dfrac{A_1 l_1}{A_2} = \dfrac{A \times l_1}{3A} = \dfrac{l_1}{3}$

$\Rightarrow\ \dfrac{l_1}{l_2} = 3$

As, $\Delta x_1 = \dfrac{F_1}{A\gamma} \times l_1$ ...(i)

and $\Delta x_2 = \dfrac{F_2}{3A\gamma} l_2$ ...(ii)

Here, $\Delta x_1 = \Delta x_2$

$\therefore\ \dfrac{F_2}{3A\gamma} l_2 = \dfrac{F_1}{A\gamma} l_1$

$\Rightarrow\ F_2 = 3F_1 \times \dfrac{l_1}{l_2} = 3F_1 \times 3 = 9F$

7 For steel wire,

$$\Delta l_s = \frac{F \times L_s}{Y_s \times A_s} = \frac{3MgL_s}{Y_s \times \pi r_s^2}$$

[$\because$ Mass (steel) $= 2M + M = 3M$]

For brass wire,

$$\Delta l_b = \frac{F \times l_b}{Y_b \times A_b} = \frac{2MgL_b}{Y_b \times \pi r_b^2}$$

[$\because$ Mass (brass) $= 2M$]

$$\Rightarrow \frac{\Delta l_s}{\Delta l_b} = \frac{3Mg\,L_s}{Y_s \times \pi r_s^2} \times \frac{Y_b \times \pi r_b^2}{2MgL_b}$$

$$= \frac{3}{2} \times \frac{L_s}{L_b} \times \frac{Y_b}{Y_s} \times \left(\frac{r_b}{r_s}\right)^2$$

$$= \frac{3a}{2b^2c}$$

8 From the definition of bulk modulus,

$$B = \frac{-dp}{(dV/V)}$$

Substituting the values, we have

$$B = \frac{(1.165 - 1.01)\times 10^5}{(10\ /\ 100)} = 1.55\times 10^5\,\text{Pa}$$

9 $\because$ Bulk modulus,

$$K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}} = \frac{\Delta p}{\dfrac{\Delta V}{V}}$$

$$\Rightarrow K = \frac{mg}{a\left(\dfrac{3\Delta r}{r}\right)}$$

$$\left[\because V = \frac{4}{3}\pi r^3,\ \text{so}\ \frac{\Delta V}{V} = \frac{3\Delta r}{r}\right]$$

$$\Rightarrow \frac{\Delta r}{r} = \frac{mg}{3aK}$$

10 If Hooke's law is being obeyed, then force extension graph is a straight line. Stored elastic energy extension $\left(U = \dfrac{1}{2}Fx = \dfrac{1}{2}\dfrac{YA}{L}x^2\right)$ should be a parabolic curve symmetric about the U axis. Hence, in the graph P represents extension and Q the stored elastic energy.

11 As work done

$$= \frac{1}{2} Y \times (\text{strain})^2 \times \text{volume}$$

$$\Rightarrow\ 2 = \frac{1}{2} \times Y \times \left(\frac{\Delta L}{L}\right)^2 \times AL$$

$$= \frac{YA(\Delta L)^2}{2L}$$

Now, work done,

$$W' = \frac{Y(4A)(\Delta L)^2}{2(L/2)}$$

$$= 8\left(\frac{YA(\Delta L)^2}{2L}\right)$$

$$= 8 \times 2 = 16\,\text{J}$$

12 Thermal stress $\sigma = Y\,\alpha\,\Delta\theta$

Given, $\sigma_1 = \sigma_2$

$\therefore\ Y_1\alpha_1\Delta\theta = Y_2\alpha_2\Delta\theta$

or $\dfrac{Y_1}{Y_2} = \dfrac{\alpha_2}{\alpha_1} = \dfrac{3}{2}$

13 If the deformation is small, then the stress in a body is directly proportional to the corresponding strain.

According to Hooke's law, i.e.

Young's modulus

$$(Y) = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

So, $Y = \dfrac{F/A}{\Delta L/L}$

$$= \frac{FL}{A\,\Delta L}$$

If the rod is compressed, then compressive stress and strain appear. Their ratio Y is same as that for tensile case.

Given, length of a steel wire (L) = 10 cm

Temperature $(\theta) = 100°\text{C}$

As length is constant.

$\therefore$ Strain $= \dfrac{\Delta L}{L} = \alpha\,\Delta\theta$

Now, pressure = stress = $Y \times$ strain

[Given, $Y = 2 \times 10^{11}\,\text{N/m}^2$ and $\alpha = 1.1 \times 10^{-5}\,\text{K}^{-1}$]

$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$

$= 2.2 \times 10^8$ Pa

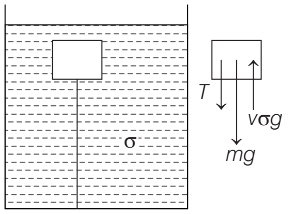
14 Change in length $\Delta L = \alpha L\,\Delta T = \dfrac{FL}{AY}$

$\Rightarrow$ Stress $= \dfrac{F}{A} = Y\alpha\Delta T = Y\alpha t$

(as, $\Delta T = t$)

15 From free body diagram of the wooden block,

$$V\sigma g = mg + T$$
[V is the volume of block]



$$T = V\sigma g - mg$$

$$T = \frac{m}{\rho}\sigma g - mg = mg\left(\frac{\sigma - \rho}{\rho}\right)$$

16 Because $\rho_{oil} < \rho < \rho_{water}$

So, it will sink through oil but will not sink in water. So, it will stay at the oil-water interface.

17 Let R be the radius of the bigger drop, then volume of bigger drop
$= 2 \times$ volume of small drop

$$\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3} r$$

Surface energy of bigger drop,

$$E = 4\pi R^2 T = 4 \times 2^{2/3}\pi r^2 T$$
$$= 2^{8/3}\pi r^2 T$$

18 We have,

$$6\pi\eta r v = \frac{4}{3}\pi r^3 g\rho - \frac{4}{3}\pi r^3 \sigma g$$

where, $\rho \to \rho_{water}$ and $\sigma \to \rho_{air}$

$$\Rightarrow \eta = \frac{2gr^2(\rho - \sigma)}{9v}$$
$$= \frac{2 \times 9.81 \times (0.2 \times 10^{-2})^2 \times 999}{9 \times 8.7}$$
$$= 1 \times 10^{-3} \text{ poise}$$

19 $v \propto \dfrac{\rho - \rho_0}{\eta}$

$$\therefore \frac{v_2}{v_1} = \frac{\rho - \rho_{02}}{\rho - \rho_{01}} \times \frac{\eta_1}{\eta_2}$$

$$v_2 = \frac{7.8 - 1.2}{7.8 - 1} \times \frac{8.5 \times 10^{-4} \times 10}{13.2}$$
$$= 6.25 \times 10^{-4} \text{ cms}^{-1}$$

20 From Bernoulli's theorem,

$$\rho g h = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$\Rightarrow gh = \frac{1}{2}v_1^2\left[\left(\frac{v_2}{v_1}\right)^2 - 1\right]$$

$$= \frac{1}{2}v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$(\because A_1 v_1 = A_2 v_2)$

$$\Rightarrow \left(\frac{A_1}{A_2}\right)^2 = 1 + \frac{2hg}{v_1^2}$$

$$\Rightarrow \left(\frac{D_1}{D_2}\right)^4 = 1 + \frac{2gh}{v_1^2}$$

$$\Rightarrow D_2 = \frac{D_1}{\left(1 + \frac{2gh}{v_1^2}\right)^{1/4}}$$

$$= \frac{8 \times 10^{-3}}{\left(1 + \frac{2 \times 10 \times 0.2}{(0.4)^2}\right)^{1/4}}$$

$$= 3.6 \times 10^{-3} \text{ m}$$

21 Velocity head $= \dfrac{v^2}{2g}$

and pressure head $= \dfrac{p}{\rho g}$

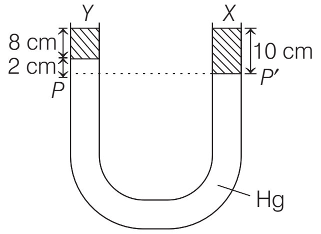
As velocity of water is equal to the pressure head of 40 cm of Hg column, hence

$$\frac{v^2}{2g} = \frac{h\rho g}{\rho g}$$
$$\Rightarrow v^2 = 2hg$$
$$\Rightarrow v = \sqrt{2gh}$$
$$= \sqrt{2 \times 9.8 \times 0.4}$$
$$= 2.8 \text{ ms}^{-1}$$

22 As shown in the figure, in the two arms of the tube the pressure remains the same on the surface PP'.

Hence,

$$8 \times \rho_Y \times g + 2 \times \rho_{Hg} \times g = 10 \times \rho_X \times g$$
$$\therefore \quad 8\rho_Y + 2 \times 13.6 = 10 \times 3.36$$



or $\rho_Y = \dfrac{33.6 - 27.2}{8} = 0.8 \text{ g / cc}$

23 At equilibrium, weight of the given block is balanced by force due to surface tension, i.e. $2L\,S = w$

$$\text{or} \quad S = \frac{w}{2\,L}$$
$$= \frac{1.5 \times 10^{-2} \text{ N}}{2 \times 0.3 \text{ m}}$$
$$= 0.025 \text{ Nm}^{-1}$$

24 Work done = Change in surface energy

$$\Rightarrow \quad W = 2T \times 4\pi(R_2^2 - R_1^2)$$
$$= 2 \times 0.03 \times 4\pi[(5)^2 - (3)^2] \times 10^{-4}$$
$$W = 0.4\,\pi \text{ mJ}$$

25 As, $T = \dfrac{r\left(h + \frac{r}{3}\right)\rho g}{2\cos\theta}$

$$= \frac{0.5 \times 10^{-2}\left[3 + \frac{0.5}{3}\right] \times 10^{-2} \times 10^3 \times 9.81}{2}$$

$$= 0.77 \text{ N/m}$$

26 (A) When a spinning ball is thrown it deviates from its usual path in flight. This is due to magnus effect.

(B) Viscous forces tends to reduce the speed of flowing fluid by virtue of internal frictional force. Hence, the energy of liquid flow reduced.

(C) Pascal's law states that if gravity effect is neglected, the pressure of every point of liquid in equilibrium of rest is the same.

(D) Gas burners are based on Bernoulli's principle.

27 The shape of the aeroplane wings is peculier i.e. its upper surface is more curved than its lower surface. Due to this, when the aeroplane runs on runway the speed of air above the wings is larger than the speed below the wings. Thus, according to Bernoulli's theorem, the pressure above wings becomes lesser than the pressure below the wings. Due to this difference of pressure a vertical lift acts on aeroplane.

28 The height of column is given by ascent formula,

$$h = \frac{2S\cos\theta}{r\rho g} - \frac{r}{3}$$

If the tube is very narrow, $r/3$ can be neglected in comparison with h.

Hence, $h = \dfrac{2S\cos\theta}{r\rho g}$.

Thus, as the value of r (radius of tube) decreases, the height increases.

$$\left(\because h \propto \frac{1}{r}\right)$$

29 A bridge during its use undergoes alternating stress and strain for a large number of times each day, depending on movement on it. With time it loses its elastic strength and the amount of strain in the bridge for a given stress becomes large and ultimately it may collapse.

30 In case of small drop of mercury the force of gravity is small and the surface tension plays a vital role. Therefore, surface tends to have minimum surface area and sphere has minimum area.

In case of large mercury drop, the gravitational pull becomes more effective than the surface tension and exerts downwards pull on the drop. Hence, the large drop of mercury becomes elliptical or oval.

Also, surface tension of a liquid decreases with rise of temperature, $S_t = S_o(1-\alpha t)$.

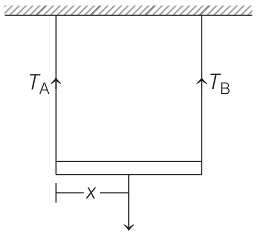
SESSION 2

1 Let tension in wire A and B be T_A and T_B, respectively,

$$T_A + T_B = W$$

and $$T_A \times x = T_B(L-x)$$

Solving the above equations,



$$T_A = \frac{W(L-x)}{L},$$

$$T_B = \frac{Wx}{L}$$

Stress in $A = \frac{T_A}{A_A}$, where A_A is cross-section area of wire A.

Stress in $B = \frac{T_B}{A_B}$, where A_B is cross-section area of wire B.

It is given,

$$A_A = \frac{A_B}{2},$$

$$\frac{T_A}{A_A} = \frac{T_B}{A_B}$$

which gives $x = \frac{2L}{3}$

2 We know that time period,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

When additional mass M is added to its bob

$$T_M = 2\pi\sqrt{\frac{L+\Delta L}{g}},$$

where, ΔL is increase in length.

We know that Young modulus of the material,

$$Y = \frac{Mg/A}{\Delta L/L} = \frac{MgL}{A\Delta L}$$

$$\Rightarrow \quad \Delta L = \frac{MgL}{AY}$$

$$\therefore \quad T_M = 2\pi\sqrt{\frac{L+\frac{MgL}{AY}}{g}}$$

$$\Rightarrow \quad \left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$$

or $$\frac{Mg}{AY} = \left(\frac{T_M}{T}\right)^2 - 1$$

or $$\frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

3 If L is the initial length, then the increase in length by a tension

F is given by $l = \frac{FL}{\pi r^2 Y}$

Hence, $a = L + l = L + \frac{4L}{\pi r^2 Y} = L + 4C$

and $\quad b = L + \frac{5L}{\pi r^2 Y} = L + 5C$

where, $C = \frac{L}{\pi r^2 Y}$

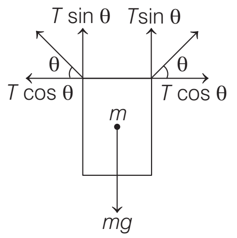
Thus, on solving for L and C, we get

$$L = 5a - 4b \text{ and } C = b - a$$

Hence, for $F = 9$ N, we get

$$x = L + \frac{9L}{\pi r^2 Y}$$
$$= L + 9C$$
$$= (5a - 4b) + 9(b - a)$$
$$= 5b - 4a$$

4 Consider the FBD diagram of the rectangular frame



Balancing vertical forces

$2T\sin\theta - mg = 0$

[T is tension in the string]

$\Rightarrow \quad 2T\sin\theta = mg \quad$ ...(i)

Total horizontal force

$$= T\cos\theta - T\cos\theta = 0$$

Now from Eq. (i), we get

$$T = \frac{mg}{2\sin\theta}$$

As, mg is constant

$$\Rightarrow T \propto \frac{1}{\sin\theta} \Rightarrow T_{max} = \frac{mg}{2\sin\theta_{min}}$$

$$\sin\theta_{min} = 0 \quad \Rightarrow \quad \theta_{min} = 0$$

No option matches with $\theta = 0°$

$$T_{min} = \frac{mg}{2\sin\theta_{max}}$$

(since, $\sin\theta_{max} = 1$)

$$\sin\theta_{max} = 1 \quad \Rightarrow \quad \theta = 90°$$

Matches with option (b).

Hence, tension is least for the case (b).

Note *We should be careful when measuring the angle, it must be in the direction as given in the diagram.*

5 Bulk modulus of rubber $(K) = 9.8 \times 10^8$ N/m^2

Density of sea water $(\rho) = 10^3$ kg/m^3

Percentage decrease in volume,

$$\left(\frac{\Delta V}{V} \times 100\right) = 0.1$$

or $$\frac{\Delta V}{V} = \frac{0.1}{100}$$

or $$\frac{\Delta V}{V} = \frac{1}{1000}$$

Let the rubber ball be taken up to depth h.

$\therefore$ Change in pressure $(p) = h\rho g$

$\therefore$ Bulk modulus

$$(K) = \frac{p}{(\Delta V/V)}$$

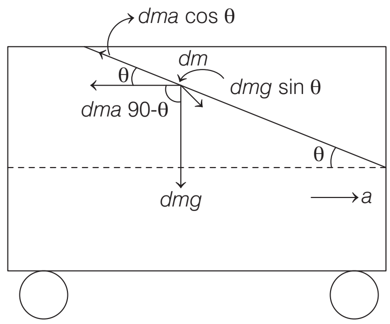
$$= \frac{h\rho g}{(\Delta V/V)}$$

or $$h = \frac{K \times (\Delta V/V)}{\rho g}$$

$$= \frac{9.8 \times 10^8 \times \frac{1}{1000}}{10^3 \times 9.8} = 100 \text{ m}$$

6 As the tanker starts accelerating free surface of the tanker will not be horizontal because pseudo force acts.

Consider the diagram where a tanker is accelerating with acceleration a.



Consider an elementary particle of the fluid of mass dm

The acting forces on the particle with respect to the tanker are shown above.

Now, balancing forces (as the particles is in equilibrium) along the inclined direction, component of weight = component of pseudo force, i.e.

$dmg\sin\theta = dma\cos\theta$ (we have assumed that the surface is inclined at an angle θ)

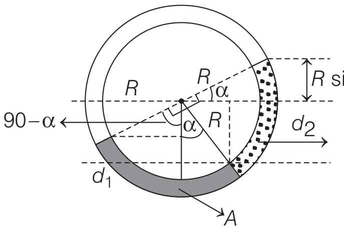
where, dma is pseudo force

$$\Rightarrow \quad g\sin\theta = a\cos\theta \Rightarrow a = g\tan\theta$$

$$\Rightarrow \quad \tan\theta = \frac{a}{g} = \text{slope}$$

7 Equating pressure at A, we get

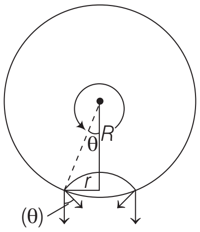
$R\sin\alpha\, d_2 + R\cos\alpha\, d_2 + R(1-\cos\alpha)\, d_1 = R(1-\sin\alpha)\, d_1$



$(\sin\alpha + \cos\alpha)\, d_2 = d_1(\cos\alpha - \sin\alpha)$

$$\Rightarrow \quad \frac{d_1}{d_2} = \frac{1+\tan\alpha}{1-\tan\alpha}$$

8 The bubble will detach if, Buoyant force $\geq$ Surface tension force



$\int \sin\theta\, T \times dl = T(2\pi r)\sin\theta$

$$\frac{4}{3}\pi R^3 \rho_w g \geq \int T \times dl \sin\theta$$

$$(\rho_w)\left(\frac{4}{3}\pi R^3\right) g \geq (T)(2\pi r)\sin\theta$$

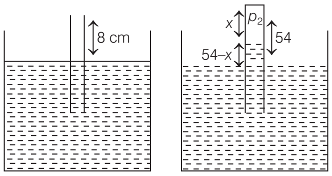
As, $\quad \sin\theta = \dfrac{r}{R}$

Solving, $r = \sqrt{\dfrac{2\rho_w R^4 g}{3T}} = R^2\sqrt{\dfrac{2\rho_w g}{3T}}$

9 In this question, the system is accelerating horizontally i.e. no component of acceleration in vertical direction. Hence, the pressure in the vertical direction will remain unaffected.

i.e. $\quad p_1 = p_0 + \rho g h$

Again, we have to use the concept that the pressure in the same level will be same.



For air trapped in tube,

$p_1 V_1 = p_2 V_2$

$p_1 = p_{atm} = \rho g 76$

$V_1 = A \cdot 8$

[where, A = area of cross-section]

$p_2 = p_{atm} - \rho g(54-x) = \rho g(22+x)$

$V_2 = A \cdot x$

$\rho g 76 \times 8A = \rho g(22+x)A\,x$

$x^2 + 22x - 76\times 8 = 0 \Rightarrow x = 16\text{ cm}$

10 Given, surface tension of water $(S) = 7.3\times10^{-2}$ N/m

Density of water $(\rho) = 1.0\times10^3$ kg/m^3

Acceleration due to gravity,

$g = 9.8$ m/s^2

Angle of contact $\theta = 0°$

Diameter of one side, $2r_1 = 3.0$ mm

$\therefore \quad r_1 = 1.5\text{ mm} = 1.5\times10^{-3}$ m

Diameter of other side

$2r_2 = 6.0$ mm

$r_2 = 3.0$ mm

$= 3.0\times10^{-3}$ m

Height of water column rises in first and second tubes

$$h_1 = \frac{2S\cos\theta}{r_1\rho g}$$

$$h_2 = \frac{2S\cos\theta}{r_2\rho g}$$

$\therefore$ Difference in levels of water rises in both tubes,

$$\Delta h = h_1 - h_2 = \frac{2S\cos\theta}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{2\times7.3\times10^{-2}\times\cos 0°}{1.0\times10^3\times9.8}\left[\frac{1}{1.5\times10^{-3}} - \frac{1}{3.0\times10^{-3}}\right]$$

$$= \frac{14.6}{9.8}\times10^{-2}\left[\frac{2-1}{3}\right]$$

$= 4.9\text{ mm} \approx 5\text{ mm}$

11 Let the densities of metal and water be ρ and ρ_0 respectively and let x be the length of the rod immersed in water at an instant of time t. Then, acceleration at that instant = apparent weight divided by the mass of the rod, i.e.

$$\frac{dv}{dt} = \frac{\pi r^2 l\rho g - \pi r^2 x\rho_0 g}{\pi r^2 l\rho}$$

$$= g - \frac{gx\rho_0}{l\rho} = g\left(1 - \frac{x}{\sigma l}\right)$$

or $\quad \dfrac{dv}{dx}\dfrac{dx}{dt} = g\left(1 - \dfrac{x}{\sigma l}\right)$

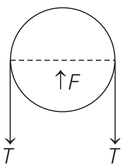
or $\quad v\dfrac{dv}{dx} = g\left(1 - \dfrac{x}{\sigma l}\right)$

On integrating, we get

$$\frac{v^2}{2} = g\left|x - \frac{x^2}{2\sigma l}\right|_0^l = g\left(l - \frac{l}{2\sigma}\right)$$

$$\Rightarrow \quad v = \sqrt{2gl\left(1 - \frac{1}{2\sigma}\right)}$$

12 Elongation due to change in temperature,



$\Delta l = L\alpha\Delta T$

which is compensated by elastic strain, when temperature becomes normal, *i.e.*,

$$\Delta l = \frac{TL}{YS}$$

Thus, $\quad \dfrac{TL}{YS} = L\alpha\Delta T$

$\Rightarrow \quad T = YS\alpha\Delta T$

At equilibrium, force exerted by one half on other,

$F = 2T = 2YS\alpha\Delta T$

13 Frequency, $f = \dfrac{v}{2l} = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}} = \dfrac{1}{2l}\sqrt{\dfrac{T}{Ad}}$

where, T is tension in the wire and μ is the mass per unit length of wire.

Also, Young's modulus,

$$Y = \frac{Tl}{A\Delta l}$$

$$\Rightarrow \quad \frac{T}{A} = \frac{Y\Delta l}{l}$$

Putting this value in expression of frequency, we have

$$f = \frac{1}{2l}\sqrt{\frac{y\Delta l}{ld}}$$

Given, $l = 1.5$ m, $\dfrac{\Delta l}{l} = 0.01$

$d = 7.7\times10^3$ kg / m^3,

$Y = 2.2\times10^{11}$ N / m^2

Putting these values we, have

$$f = \frac{1}{2l}\sqrt{\frac{2.2\times10^{11}\times0.01}{7.7\times10^3}}$$

$$f = \sqrt{\frac{2}{7}}\times\frac{10^3}{3}$$

$f \approx 178.2$ Hz

Hence, option (b) is true.

14 As, $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{F}{\pi r^2\times\text{strain}}$

$\Rightarrow \quad F = T = Y\pi r^2\times\text{strain}$

Now, strain

$$= \frac{\sqrt{L^2+l^2} - L}{L} = \left(1 + \frac{l^2}{L^2}\right)^{1/2} - 1$$

$$\approx 1 + \frac{1}{2}\frac{l^2}{L^2} - 1 = \frac{l^2}{2L^2}$$

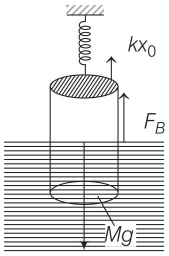
(by binomial expansion)

$$\therefore \; T = \frac{Y\pi r^2\cdot l^2}{2L^2}$$

15 In equilibrium,

upward force = downward force

$kx_0 + F_B = mg$



Here, kx_0 is restoring force of spring and F_B is buoyancy force.

$$kx_0 + \sigma \frac{L}{2} Ag = Mg$$

$$x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{\sigma LA}{2M}\right)$$

16 Given, $Y = 2 \times 10^{11}$ N/m^2

Stress = 5×10^7 N / m^2

As, $\frac{\text{stress}}{\text{strain}} = Y$

$\Rightarrow$ Strain = $\frac{5 \times 10^7}{2 \times 10^{11}} = 2.5 \times 10^{-4}$

It is symmetric strain.

Now, strain of 2.5×10^{-4} is equivalent.

As, $\frac{\Delta V}{V} = 3\left(\frac{\Delta r}{r}\right)$

$\therefore$ $\frac{2.5 \times 10^{-4}}{3}$

$= \frac{\Delta r}{r} = 0.83 \times 10^{-4}$

$\therefore$ Fraction decrease in radius

$= (1.00 - 0.83)\, 10^{-4}$

$= 0.17 \times 10^{-4}$

17 Here, $Y_c = 1 \times 10^{11}$ N/m^2

$Y_s = 2 \times 10^{11}$ N/m^2

$l_c = 1.0$ m, $l_2 = 0.5$ m

and $\Delta l_c = 1 \times 10^{-3}$ m

As, $(\text{strain})_c = \frac{\text{stress}}{Y_c}$

$\Rightarrow$ $1 \times 10^{-3} = \frac{\text{stress}}{1 \times 10^{11}}$

$\Rightarrow$ stress = 10^8 N/m^2

Now, $Y_s = \frac{\text{stress}}{\text{strain}}$

$\Rightarrow$ strain = $\frac{10^8}{2 \times 10^{11}} = 0.5 \times 10^{-3}$

or $\frac{\Delta l_s}{1/2} = 0.5 \times 10^{-3}$

$\Rightarrow$ $\Delta l_s = 0.25 \times 10^{-3}$

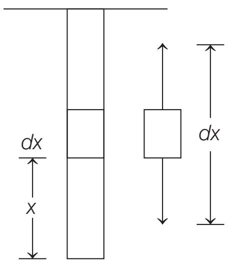
$\therefore$ $\Delta l = \Delta l_c + \Delta l_s = 1 + 0.25$

$= 1.25$ mm

18 Consider an element of wire of width dx at a distance x from bottom end of wire. The force experienced by this element is due to the gravitational force of portion of wire lower to it.

So, $Y = \frac{T/A}{\frac{\Delta(dx)}{dx}}$, where $\Delta(dx)$ is the elongation in this element.

Now, $\Delta(dx) = \frac{T}{YA}dx = \frac{mg}{YAl} \times xdx$



Total elongation,

$$\Delta l = \int \Delta(dx)$$

$$= \int_0^l \frac{mg \times xdx}{YAl}$$

$$= \frac{mgl}{2YA}.$$

19 The density of concrete of course, is more than that of water and a block of concrete will sink like a stone, if dropped into water. Concrete cargo were filled with air and as such, average density of cargo vessels

$$= \frac{[\text{Mass of concrete} + \text{Mass of air}]}{[\text{Volume of concrete} + \text{Volume of air}]}$$

It follows that the average density of cargo vessels must be less than that of water. As a result, the concrete cargo vessels did not sink.

20 From the continuity equation,

Av = constant

where, Av is the volume of liquid flowing per second.

or $A \propto \frac{1}{v}$

As the stream falls, its speed v increases and hence its area of cross-section a will decrease. That is why the stream will become narrow.

When the stream will go up, its speed decreases, hence its area of cross-section will increase and it will become broader and spreads out like a fountain. Hence, option (a) is true.